Additional Comments on Holographic Superconductor (with Momentum Relaxation[1])

In collaboration with
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[1] arXiv:1501.00446

What is Superconductor?

1. Superconductivity was discovered in 1911 in an attempt to understand how the resistivity of a metal behaves at low temperature.
   - Zero Electrical Resistivity below a critical temperature
   - Missner effect : a magnetic field is expelled when $T < T_c$
2. Landau-Ginzburg theory : describes superconductivity in terms of a second order phase transition whose order parameter is a complex scalar field φ

$$\frac{dF}{d\varphi^*} = \alpha(T - T_c)\varphi + \beta|\varphi|^2\varphi + \cdots$$

   where α and β are positive constants
   - $T > T_c$: the min. of the free energy is at $\varphi = 0$
   - $T < T_c$: the min. of the free energy is at nonzero value of φ
3. BCS theory
   - Cooper pairs by electron-phonon interaction
   - Pairing state is singlet and s-wave
   - Cooper pair and BEC $\Rightarrow$ (universal) energy gap, Δ

Why Holographic Superconductor?

Unconventional Superconductor

1. Cuprates($T_c = 164K$) were discovered in 1986 (The highest known T_c is $23K$ for conventional superconductor)
2. Pnitides was found in an iron arsenide compound in 2008, and superconductivity was seen up to $56K$
3. Electron pairing state is singlet and d-wave
4. The lattice should not be ignored (strongly correlated)
5. Anisotropy of energy gap in momentum space

- There is no known mechanism for high temperature superconductor.
- Can we find a phenomenological model first such as Landau-Ginzberg model for high temperature superconductor?
- Let us try this with another view point, which is using gravity, holographic correspondence conjecture.

Holographic Superconductor by HHH

$$S = \int d^4x \sqrt{-g}\left(R - 2\Lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\nabla_\mu \psi - iqA_\mu \psi|^2 - m^2\psi^2\right) \quad (1)$$

where q is charge of scalar field.

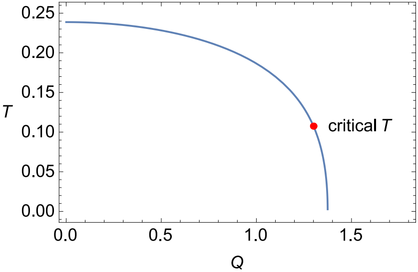
Holographic Correspondence (AdS/CMT)

1. RN-AdS black hole in gravity $\Leftrightarrow$ normal states in CMT
2. RN-Hairy black hole in gravity $\Leftrightarrow$ superconducting phase in CMT
3. critical temperature, T_c, which phase transition occur
4. Scalar field Condensation, $\langle \mathcal{O} \rangle$
5. Energy gap, Δ. The gap in the spectrum results in a gap in the optical conductivity, i.e. the conductivity as a function of frequency, $\Delta = 2\omega_g$

Schwinger Mechanism at black hole horizon

: Basic Mechanism for forming Hairy Black Hole from RN-AdS black hole suggested by Gary Horowitz

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2), \quad \text{where} \quad f(r) = r^2 - \frac{M}{r} + \frac{Q^2}{4r^2}$$



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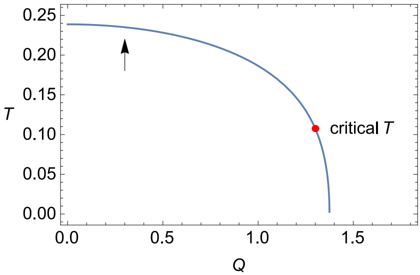

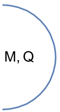

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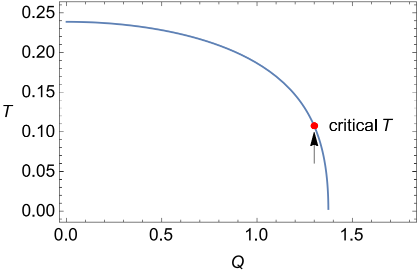

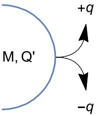

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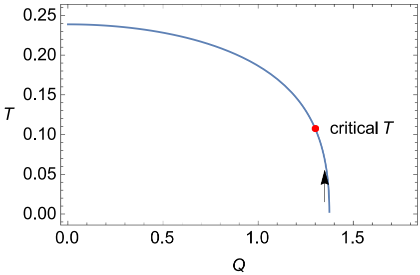


BF bound

For RN-AdS black hole, the effective mass of ψ is

$$m_{eff}^2 = m^2 + q^2 g^{tt} A_t^2$$

- m_{eff}^2 can be sufficiently negative near the horizon to destabilize the scalar field → The origin of the instability responsible for the scalar hair is the coupling of the charged scalar to the charge of the black hole.
- AdS_{d+1} spacetime is stable even with scalar fields with $m^2 < 0$ provided $m^2 > m_{BF}^2$ with $m_{BF}^2 = -\frac{d^2}{4L^2}$, (We consider the case $m^2 = -\frac{2}{L^2}$)

Would the instability turn off as $q \to 0$?

- A nearly extremal RN-AdS black hole remains unstable to forming neutral scalar hair, provided that m^2 is close to the BF bound
- near horizon geometry of an extremal RN-AdS black hole is $AdS_2 \times R^2$
- If $m_{BF,AdS_4}^2 < m^2 < m_{BF,AdS_2}^2$, it is unstable at near horizon, but stable asymptotic AdS_4
- This process is not associated with superconductivity since it does not break $U(1)$ symmetry

Qausi-normal Mode

- The Chebyshev grid and Chebyshev differential matrix are used
- Temperature of RN-AdS black hole is calculated by

$$T = \frac{12L^2 - Q^2 z_+^4}{16L^2 \pi z_+} \tag{2}$$

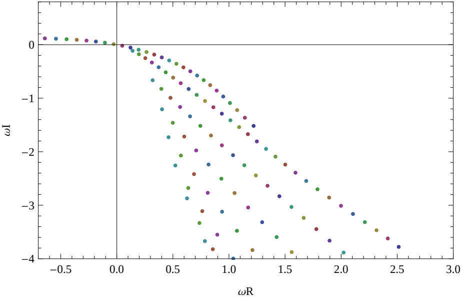


Figure: Q changes from 0.01 to 3 with increment of 0.1.

Free energy comparison

Grand potential is

$$F = \epsilon - Ts - \mu\rho$$

where ϵ is energy density, ρ is charge density, and s is entropy density.

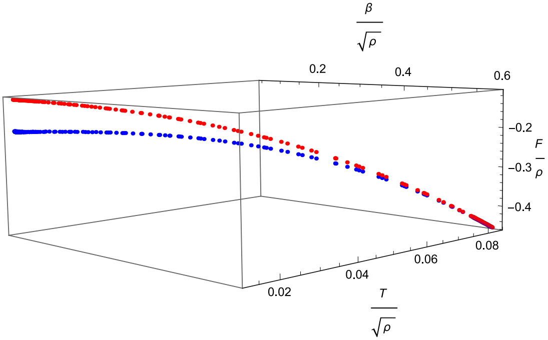


Figure: Red is for RN-AdS black hole and Blue is for hairy black hole when ρ is fixed.

Setup for Momentum Relaxation

$$S = \int_M d^{d+1}x\sqrt{-g}\left[R + \frac{d(d-1)}{L^2} - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2 - \frac{1}{2}\sum_{I=1}^{d-1}(\partial\psi_I)^2\right]$$

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + \frac{r^2}{L^2}(dx^2 + dy^2)\,,$$

$$A = A_t(r)dt\,, \qquad \Phi = \Phi(r)\,, \qquad \psi_I = \beta_{Ii}x^i = \frac{\beta}{L^2}\delta_{Ii}x^i\,,$$

Condensation

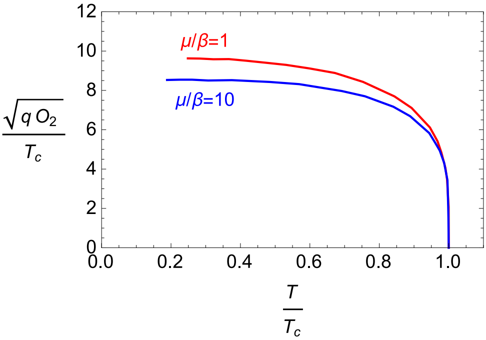
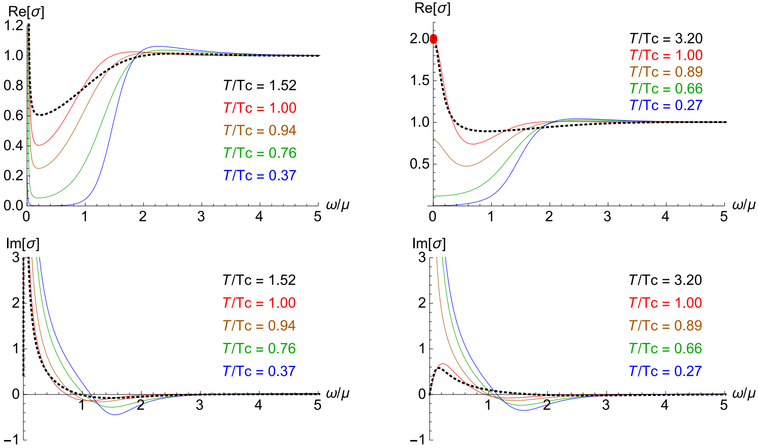
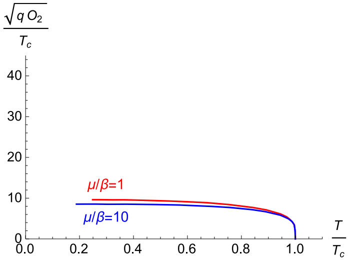


Figure: condensation for $\mu/\beta = 1$ for Red and $\mu/\beta = 10$ for Blue with $\Delta = 2$ and $q = 3$

Conductivity



New Mechanism for Holographic Superconductor



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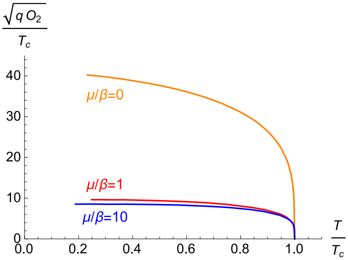

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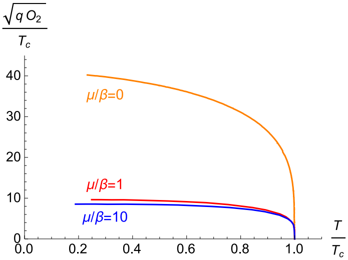

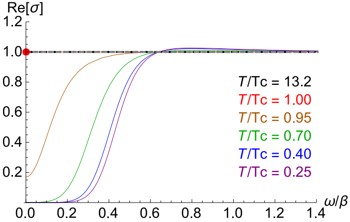

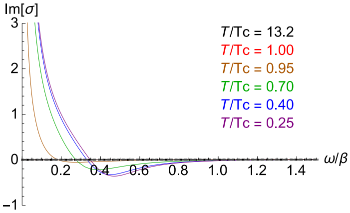

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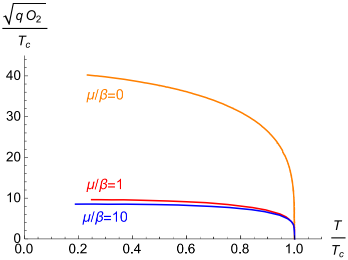

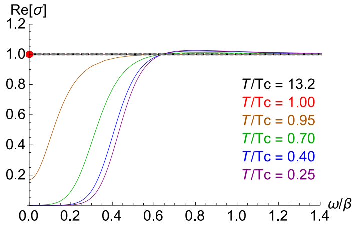

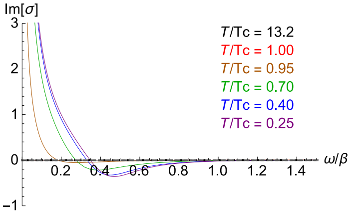

But physical mechanism is not understood yet!